

Wigner’s Friend as Contextual Gluing: A Categorical and Homotopy-Type-Theoretic Formalization of Observer-Relative Quantum Facts

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Wigner’s friend is usually posed as a conflict between a definite internal measurement record and an external unitary description of the sealed laboratory. This paper reformulates the problem as a categorical and homotopy-type-theoretic gluing problem. Physical-observational situations are represented as objects of a context category $\mathcal{C}$. Quantum state assignment is represented by a presheaf $Q : \mathcal{C}^{\text{op}} \rightarrow \text{St}$, while record assignment is represented by a presheaf $R : \mathcal{C}^{\text{op}} \rightarrow \text{Type}$. Measurement is modeled not as primitive global collapse but as a morphism $m : S \rightarrow F$ producing a record-bearing context. The friend’s outcome is a typed witness $\Gamma_F \vdash p_i : \text{Saw}(F, i)$, whereas Wigner’s pre-communication description is a different judgment $\Gamma_W \vdash \Psi_L : \text{LabState}$. The apparent contradiction results from erasing context and inferring $\Gamma_W \vdash r_F = i$ without a transporting morphism. Communication creates a joint refinement J , with comparison maps $\pi_F : J \rightarrow F$ and $\pi_W : J \rightarrow W$. Public facthood is then defined as inhabitation of a gluing type $\text{Glue}_J(r_F, r_W) := (\pi_F^*(r_F) = \pi_W^*(r_W))$. The obstruction is therefore not to local definiteness but to replacing a context-indexed state-record diagram by one Boolean global section. The paper concludes that quantum reality is best represented as an equivalence class of glueable state-record diagrams rather than as a context-free classical fact table.

Keywords: Wigner’s friend, quantum foundations, category theory, homotopy type theory, HoTT, univalence, quantum contextuality, presheaf, sheaf, global section, relational quantum mechanics, observer-relative facts, quantum measurement.

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1. Introduction

Wigner’s friend exposes a structural ambiguity in the phrase “the quantum state of a system.” Inside a sealed laboratory, the friend measures a quantum system and obtains a definite result. Relative to the friend’s post-measurement situation, there is a record. The apparatus, memory, and local environment are correlated with one outcome. Outside the sealed laboratory, Wigner may consistently assign a unitary quantum state to the entire lab, including the friend and the friend’s record. The friend’s description is outcome-specific. Wigner’s description is branch-

inclusive. The problem is not merely epistemic disagreement. The two descriptions are licensed by different physical relations to the same experiment.

This paper proposes that the paradox is produced by insufficient typing of physical claims. The friend's judgment belongs to one context; Wigner's judgment belongs to another. The friend has an inhabited record type. Wigner has an external laboratory-state judgment. These are not contradictory unless they are forced into a single untyped arena. The relevant mathematical question is whether local state-record assignments glue into a common refinement when communication occurs.

The guiding thesis is:

Quantum reality is not a context-free table of Boolean facts. It is an equivalence-governed diagram of state assignments, records, witnesses, and context morphisms.

The formal replacement is:

Reality $\neq \Psi_{\text{univ}}$

Reality = $[Q, R, \mathcal{C}, \text{transport}, \text{glue}]_{\simeq}$

Here Q is a quantum-state presheaf, R is a record presheaf, $\mathcal{C}$ is a category of physical-observational contexts, and $\simeq$ denotes admissible physical equivalence.

2. Context Category

Let $\mathcal{C}$ be a category of physical-observational contexts.

An object $C \in \text{Obj}(\mathcal{C})$ is a structured context:

$C := (H_C, A_C, R_C, I_C)$

where H_C is the Hilbert space or effective system algebra visible in context C , A_C is the admissible observable algebra, R_C is the record structure available in C , and I_C is the interaction history defining C .

A morphism $f : C \rightarrow D$ is an admissible physical transition between contexts. Such morphisms include preparation, measurement, inclusion, restriction, communication, coarse-graining, refinement, and updating.

Composition encodes physical concatenation:

$C \xrightarrow{f} D \xrightarrow{g} E$

$g \circ f : C \rightarrow E$

Identity morphisms encode unchanged contexts:

$$\text{id}_C : C \rightarrow C$$

For Wigner's friend, use four principal contexts:

S = prepared microscopic system

F = friend's post-measurement context

W = Wigner's sealed-lab external context

J = joint Wigner-friend communication context

The laboratory boundary is encoded categorically by absence of a record-transport morphism from F to W before communication.

Before communication:

$$F \nrightarrow W$$

After communication:

$$F \rightarrow J$$

$$W \rightarrow J$$

Thus the sealed lab is not merely a physical box. It is a missing morphism in the context diagram.

3. Quantum State Assignment as a Presheaf

Let St be a target category of state objects. For ordinary quantum mechanics, St may be Conv , the category of convex state spaces. For a HoTT-level formulation, St may be Type .

Define a quantum-state presheaf:

$$Q : \mathcal{C}^{\text{op}} \rightarrow \text{St}$$

$$C \mapsto Q(C)$$

For density operators:

$$Q(C) := D(H_C)$$

For pure states:

$$Q(C) := P(H_C)$$

where $P(H_C)$ is projective Hilbert space.

For every context morphism $f : C \rightarrow D$, the presheaf gives a restriction or re-expression map:

$$Q(f) : Q(D) \rightarrow Q(C)$$

Equivalently:

$$f^* : Q(D) \rightarrow Q(C)$$

A quantum state is therefore not assigned to a bare system but to a system-in-context:

$$\rho_C \in Q(C)$$

A compatible global quantum assignment would be a natural transformation:

$$1 \Rightarrow Q$$

equivalently a family $(\rho_C)_C$ satisfying:

$$f^*(\rho_D) = \rho_C$$

for every $f : C \rightarrow D$.

Define separately a Boolean record-valuation presheaf:

$$B : \mathcal{C}^{op} \rightarrow \text{Set}$$

where $B(C)$ is the set of Boolean record-valuations available in C .

A classical global fact table is:

$$\Gamma(B) := \text{Nat}(1, B)$$

The Wigner-friend problem is not the absence of local records. It is the illicit assumption that local records must already define a context-free Boolean global section.

4. Minimal Wigner-Friend Model

Let the measured system be:

$$H_S := \text{span}\{|0\rangle, |1\rangle\}$$

Let the initial state be:

$$|\psi\rangle_S = \alpha|0\rangle + \beta|1\rangle$$

with:

$$\alpha\beta \neq 0$$

Let the friend's ready state be:

$$|F_*\rangle$$

Let the friend-record states be:

$|F_0\rangle$ and $|F_1\rangle$

with:

$$\langle F_0 | F_1 \rangle = 0$$

The friend's measurement is a unitary correlation map:

$$U_F : H_S \otimes H_F \rightarrow H_S \otimes H_F$$

defined by:

$$U_F(|0\rangle|F_0\rangle) = |0\rangle|F_0\rangle$$

$$U_F(|1\rangle|F_0\rangle) = |1\rangle|F_1\rangle$$

Therefore:

$$U_F(|\psi\rangle|F_0\rangle) = \alpha|0\rangle|F_0\rangle + \beta|1\rangle|F_1\rangle$$

Set:

$$|\Psi_L\rangle := \alpha|0\rangle|F_0\rangle + \beta|1\rangle|F_1\rangle$$

For the friend after measurement:

$$\Gamma_F \vdash r_F : \mathbb{Z}$$

and, for some $i \in \{0,1\}$:

$$\Gamma_F \vdash r_F = i$$

For Wigner before communication:

$$\Gamma_W \vdash \Psi_L : P(H_S \otimes H_F)$$

The invalid inference is:

$$\Gamma_F \vdash r_F = i$$

therefore:

$$\Gamma_W \vdash r_F = i$$

This is invalid because Γ_F and Γ_W are different contexts. A witness in Γ_F is not automatically available in Γ_W .

5. Measurement as Record-Producing Morphism

A projective measurement with outcomes $i \in \{0,1\}$ has projectors:

$$P_i := |i\rangle\langle i|$$

The associated ideal instrument is:

$$I_i(\rho) := P_i \rho P_i$$

The probability of outcome i is:

$$p_i := \text{Tr}(I_i(\rho))$$

The conditional state is:

$$\rho_i := I_i(\rho) / \text{Tr}(I_i(\rho))$$

Categorically, measurement is a morphism:

$$m : S \rightarrow F$$

This morphism creates a record-bearing context.

Let R be a record presheaf:

$$R : \mathcal{C}^{\text{op}} \rightarrow \text{Type}$$

with:

$$R(F) = \mathbb{Z}$$

and:

$$r_F \in R(F)$$

The friend's fact is represented as an inhabited type:

$$\text{Saw}_F(i)$$

with witness:

$$p_i : \text{Saw}_F(i)$$

Before communication, Wigner does not have this witness:

$$\Gamma_W \not\vdash p_i : \text{Saw}_F(i)$$

Instead, Wigner has:

$$\Gamma_W \vdash \Psi_L : P(H_L)$$

where:

$$H_L := H_S \otimes H_F$$

Thus measurement creates local facthood in F. It does not by itself create immediate Boolean facthood in W.

6. Communication as Common Refinement

Let J be the joint context after Wigner opens the laboratory.

There are comparison maps:

$$\pi_F : J \rightarrow F$$

$$\pi_W : J \rightarrow W$$

Schematic context square:

$$J \rightarrow F$$

$$\downarrow \downarrow$$

$$W \rightarrow L$$

where L is the laboratory event-context.

Let Wigner's ready state be:

$$|W_*\rangle$$

Let Wigner's record states be:

$$|W_0\rangle \text{ and } |W_1\rangle$$

with:

$$\langle W_0 | W_1 \rangle = 0$$

The communication unitary satisfies:

$$U_W(|i\rangle|F_i\rangle|W_*\rangle) = |i\rangle|F_i\rangle|W_i\rangle$$

Therefore:

$$U_W(|\Psi_L\rangle|W_*\rangle) = \alpha|0\rangle|F_0\rangle|W_0\rangle + \beta|1\rangle|F_1\rangle|W_1\rangle$$

Set:

$$|\Psi_J\rangle := \alpha|0\rangle|F_0\rangle|W_0\rangle + \beta|1\rangle|F_1\rangle|W_1\rangle$$

Inside a record-conditioned joint context:

$$r_F = i$$

$$r_W = i$$

The shared-record condition is:

$$\pi_F^*(r_F) = \pi_W^*(r_W)$$

inside $R(J)$.

Thus public facthood is not primitive. It is an invariant in a common refinement.

7. HoTT Encoding of Observer Facts

Contexts are typing environments.

Friend context:

$$\Gamma_F$$

Wigner context:

$$\Gamma_W$$

Joint context:

$$\Gamma_J$$

The friend's record is a typed judgment:

$$\Gamma_F \vdash r_F : \mathbb{Z}$$

The proposition that the friend saw outcome i is a type:

$$\text{Saw}(F, i) : \mathcal{U}$$

The friend's evidence is an inhabitant:

$$\Gamma_F \vdash p_i : \text{Saw}(F, i)$$

Wigner's external laboratory description is a different judgment:

$$\Gamma_W \vdash \Psi_L : \text{LabState}$$

with:

$$\text{LabState} \simeq P(H_L)$$

There is no judgmental equality:

$$\text{Saw}(F, i) \not\equiv \text{LabState}$$

Therefore, the apparent contradiction is produced by type erasure.

Communication extends context:

$$\Gamma_W \rightsquigarrow \Gamma_J$$

and transports the friend witness:

$$\text{transport}_{F \rightarrow J}(p_i) : \text{Saw}_J(F, i)$$

Wigner's new witness is:

$$q_i : \text{Heard}_J(W, F, i)$$

Define:

$$\text{Consistent}_J(i) := \text{Saw}_J(F, i) \times \text{Heard}_J(W, F, i) \times (r_F = r_W)$$

A shared fact is an inhabitant:

$$c_i : \text{Consistent}_J(i)$$

Thus a public fact is not merely a Boolean value. It is a coherent witness structure in an extended context.

8. Gluing Type for Public Facthood

Let:

$$r_F \in R(F)$$

$$r_W \in R(W)$$

A shared record exists over J iff their pullbacks agree:

$$\pi_{F^*}(r_F) = \pi_{W^*}(r_W)$$

Define the gluing type:

$$\text{Glue}_J(r_F, r_W) := (\pi_{F^*}(r_F) = \pi_{W^*}(r_W))$$

When $\text{Glue}_J(r_F, r_W)$ is inhabited, the records glue.

Equivalently, shared facthood is:

$$\Sigma_{\{r_J : R(J)\}} (r_J = \pi_{F^*}(r_F)) \times (r_J = \pi_{W^*}(r_W))$$

If:

$$\text{Glue}_J(r_F, r_W) \simeq \emptyset$$

then no shared fact exists without adding a physical explanation: error, disturbance, memory revision, record erasure, deception, decoherence, reversal, or another morphism modifying the diagram.

This is the central formal proposal:

Public facthood = inhabitation of a record-gluing type.

9. Univalence and Physical Identity

Univalence asserts:

$$(A \simeq B) \rightarrow (A = B)$$

Physical reading:

Equivalent descriptions count as identical physical content when the equivalence preserves the admissible structure.

For quantum states, global phase equivalence is:

$$|\psi\rangle \sim e^{i\theta} |\psi\rangle$$

The physical pure state is:

$$[|\psi\rangle] \in P(H)$$

with:

$$P(H) := \{|\psi\rangle \neq 0\} / \mathbb{C}^\times$$

Gauge equivalence is schematically:

$$A \sim A + d\lambda$$

Basis equivalence is:

$$|\psi\rangle \sim U|\psi\rangle$$

when U preserves the observable structure.

Thus physical identity is not syntactic identity:

$$\psi = \phi$$

but equivalence-invariant identity:

$$\psi \simeq_{\text{phys}} \phi$$

In Wigner's friend:

$$r_F \neq \Psi_L$$

The friend's record and Wigner's external state are not syntactically identical. They may nevertheless belong to a single physical diagram if there exists admissible transport and gluing:

$$(r_F, \Psi_L, r_W) \in \text{Glue}(F, W, J)$$

The object of ontology is therefore not a single expression. It is the equivalence class of compatible descriptions.

10. No Natural Branch Selector

Let $P(H)$ be projective Hilbert space with $\dim H \geq 2$.

A context-free deterministic branch selector would be a map:

$$b : P(H) \rightarrow \mathbb{Z}$$

natural under all unitary equivalences:

$$b([U\psi]) = b([\psi])$$

for every $U \in U(H)$.

But $U(H)$ acts transitively on $P(H)$. Therefore b must be constant.

If:

$$b([0]) = 0$$

and:

$$b([1]) = 1$$

then b is nonconstant.

Contradiction.

Therefore, there is no basis-free natural deterministic branch selector.

A branch becomes selectable only after extra contextual structure is supplied:

$$(H, \{|0\rangle, |1\rangle\}, R_F, m)$$

Thus local record selection is allowed:

$$\Gamma_F \vdash r_F : \mathbb{Z}$$

but context-free selection is not:

$$\not\vdash b(\Psi_L) : \mathbb{Z}$$

without a context supplying the record basis and witness.

This supports the central point: outcome-selection is contextual, not basis-free and global.

11. Global Sections and Contextual Obstruction

Let M be a measurement cover.

Let:

$$V : M^{\text{op}} \rightarrow \text{Set}$$

assign to each measurement context M the set of admissible Boolean valuations:

$$V(M) := \{v_M : M \rightarrow \{0,1\}\}$$

For an orthogonal resolution:

$$\sum_i P_i = I$$

a valuation must satisfy:

$$\sum_i v(P_i) = 1$$

A global valuation is:

$$v \in \Gamma(V)$$

where:

$$\Gamma(V) := \text{Nat}(1, V)$$

Quantum contextuality is the obstruction:

$$\Gamma(V) = \emptyset$$

for sufficiently rich measurement covers.

Wigner's friend is an observer-record analogue of this failure.

Local friend valuation:

$$v_F \in V(F)$$

External Wigner state assignment:

$$\Psi_L \in Q(W)$$

Joint record valuation:

$$v_J \in V(J)$$

The forbidden move is to assume:

$$v_{\text{global}} \in \Gamma(V)$$

such that:

$$v_{\text{global}}|_F = v_F$$

$$v_{\text{global}}|_W = v_W$$

$$v_{\text{global}}|_J = v_J$$

before the morphisms that make these restrictions meaningful.

The categorical moral is:

Local sections need not determine a global classical section.

12. Main Proposition

Let $\mathcal{C}_{\text{WF}}$ be the Wigner-friend context diagram with objects F, W, J , and L , and morphisms:

$$J \rightarrow F$$

$$J \rightarrow W$$

$$F \rightarrow L$$

$$W \rightarrow L$$

Let:

$$R : \mathcal{C}_{\text{WF}}^{\text{op}} \rightarrow \text{Type}$$

be the record presheaf.

Let:

$$r_F \in R(F)$$

$$r_W \in R(W)$$

Then r_F and r_W define a shared public fact over J iff the type:

$$\text{Glue}_J(r_F, r_W) := (\pi_F^*(r_F) = \pi_W^*(r_W))$$

is inhabited.

Equivalently, shared facthood is the dependent sum:

$$\Sigma_{\{r_J : R(J)\}} (r_J = \pi_F^*(r_F)) \times (r_J = \pi_W^*(r_W))$$

Proof sketch.

A record in F is local to Γ_F . A record in W is local to Γ_W . A record in J must restrict compatibly along π_F and π_W . Thus a shared record r_J exists exactly when the pullbacks of r_F and r_W into $R(J)$ agree. In HoTT, equality of these pullbacks is itself a type. Inhabitation of this type is precisely the witness that the records glue. If the type is empty, no shared fact exists in J without additional physical morphisms changing the diagram.

13. Interpretive Consequences

First, Wigner's friend is not a contradiction between two facts. It is an invalid context-erasing inference.

Second, the friend's outcome is locally definite:

$$\Gamma_F \vdash r_F = i : \mathbb{Z}$$

Third, Wigner's external assignment is also valid in its context:

$$\Gamma_W \vdash \Psi_L : P(H_L)$$

Fourth, before communication there is no required judgment:

$$\Gamma_W \vdash r_F = i : \mathbb{Z}$$

Fifth, after communication there may be:

$$\Gamma_J \vdash r_J = i : \mathbb{Z}$$

with:

$$r_J = \pi_F^*(r_F) = \pi_W^*(r_W)$$

Therefore, the quantum state is not a classical container of observer-independent facts. It is a context-indexed section whose physical content is determined by admissible morphisms, typed witnesses, equivalences, and gluing.

14. Conclusion

The Wigner-friend problem is most naturally expressed as a failure of global classical completion. The friend has a local record. Wigner has an external state assignment. The joint context may support a shared record after communication. But these are typed judgments in different contexts, not three untyped propositions in one Boolean arena.

The formal replacement is:

$$\text{Reality} = [Q, R, \mathcal{C}, \text{transport}, \text{glue}]_{\simeq}$$

Equivalently:

Quantum reality = equivalence class of gluable state-record diagrams.

The obstruction is:

$$\Gamma(V) = \emptyset$$

when one demands a context-free Boolean global section.

Thus Wigner's friend is not primarily a paradox about consciousness, nor merely a dispute between interpretations. It is a categorical-HoTT lesson: observer facts are local witnesses, public facts are gluing types, and physical reality is the equivalence-invariant structure of the whole diagram.

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